Reg.No. \_\_\_\_\_\_\_\_\_\_\_\_

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**End Semester Examination – Nov/Dec – 2018**

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| **Code :** | **09MA314** | **Duration :** | **3hrs** |
| **Sub. Name :** | **THEORY OF NEAR-RINGS** | **Max. marks :** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

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| **Q. No.** | **Sub Div.** | **Questions** | **Course**  **Outcome** | **Marks** |
| 1. | a. | If *I* is an ideal of *N,* then prove that *N/I* is a homorphic image of *N.*  Conversely, if *h: N → N’* is an epimorphism, then prove that *Ker h* is an ideal of *N* and *N/ Ker h* and *N’* are isomorphisim. | CO1 | 12 |
| b. | Let  with  Then prove that the left ideal generated by *R* is an ideal of *N.* | CO1 | 8 |
| (OR) | | | | |
| 2. | a. | State and prove the *second isomorphism theorem.* | CO1 | 12 |
| b. | If *N* has near-ring of left quotients and , then prove that and are isomorphic. | CO1 | 8 |
|  |  |  |  |  |
| 3. | a. | Let *N* be a near-ring and *B* a subset of *N.* Then prove that *B* forms a base if and only if *(0:B) = {0}.* | CO1 | 5 |
| b. | Let *𝛑* be the natural epimorphism. Then prove that *()* forms a base for *(N).* | CO1 | 15 |
| (OR) | | | | |
| 4. | a. | Let *N* be a near-ring. Then prove that   1. *N* is abelian and Commutative if and only if *N* is a commutative ring 2. *N* is abelian and distributive if and only if *N* is a ring 3. If *N2= N* and *N* is distributive, then *N* is a ring. | CO1 | 12 |
| b. | Let *N* be a near-ring. Then   1. *n εN* is right cancellable if and only if *n* is not a right zero-divisor 2. If *n ε N0* is left cancellable, then *n* is not a left zero-divisor | CO1 | 8 |
|  |  |  |  |  |
| 5. | a. | Prove that the sum of ideals of a near-ring *N* is again an ideal of *N.* | CO1 | 6 |
| b. | For each family *{Ik}kεK* of ideals of *N.* Then prove that the following conditions are equivalent:   1. The sum of the *Ik’s* is direct. 2. The sum of the normal subgroups *(Ik, +)* is direct 3. For all *k* in *K*: *= {0}.* | CO1 | 14 |
| (OR) | | | | |
| 6. | a. | Define *decomposable near-ring.* | CO1 | 4 |
| b. | If *I* is an ideal of *N* and *N* has *DCCI,* then prove that *N/I* has *DCCI.* | CO1 | 8 |
| c. | Let *N* have the *DCCI.* Then N is the finite direct sum of indecomposable ideals. | CO1 | 8 |
|  |  |  |  |  |
| 7. | a. | Let *P* be an ideal of a near-ring *N.* Then prove that the following conditions are equivalent:   1. *P* is a prime ideal. 2. For nay ideals *I, J* of *N*, whenever *(IJ) P* implies *IP* or *J P.* 3. For any *i,j* in *N,* whenever *iP* and *jP*, we have *(i)(j)* 4. For any ideals *I* and *J* of *N,* whenever and , we have IJ 5. For any ideals *I* and *J* of *N,* whenever and , we have IJ | CO1 | 15 |
| b. | Prove that if *N* is simple, then either *N* is prime or *N* is zero near-ring. | CO1 | 5 |
| (OR) | | | | |
| 8. | a. | Let *MN* be a non-void m-system in *N* and *I* be an ideal of *N* with Then prove that *I* is contained in a prime ideal *P≠N* with | CO1 | 12 |
| b. | Let S be an sp-system and s in S. Then prove that there is some m-system M with sεM*S.* | CO1 | 8 |
|  | |  |  |  |
|  | | **Compulsory**: |  |  |
| 9. |  | Let *N* be a near-ring. Then prove that   1. If *M≤N* and *M* is distributively generated near-ring, then *N* is not distributively generated near-ring. 2. If *M≤N* and *N* is distributively generated near-ring, then *M* is not distributively generated near-ring. 3. Every homomorphic image of a distributively generated near-ring is distributively generated near-ring. 4. Every direct sum of distributively generated near-ring’s is a distributively generated near-ring. 5. Every direct summan of a distributively generated near-ring is itself distributively generated near-ring. | CO1 | 20 |